

Development of a Circuit-theory Based Transmission Cost Allocation Method by Orthogonal Projection and Equal-sharing Principle

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Abstract—This paper presents a new solution to the problem of transmission cost allocation to its users. The proposed technique utilizes modified Z-bus theory, equivalent current injection and impedance of the generators and loads, and is developed by the equal-sharing as well as the orthogonal projection principle. The procedure is performed in three steps. First, the modified Z-bus theory is used to calculate the contribution of the users into the network bus voltages as well as the branch currents. Then, the equal sharing principle is confirmed by the game theory solutions and is subsequently applied to identify the users' contributions into the branch power flows. After that, the orthogonal projections of the contributions are calculated and the concept of effective contributions is suggested. The proposed methodology provides the percentage shares of the users on the network complex variables, which help to better assess the contributions. A 2-bus and the IEEE 30-bus test system are used to validate the proposed technique. Finally, the proposed methodology is applied to the polish 2383-bus system to emphasize its applicability to large practical systems.

Index Terms—Effective contribution, equal sharing of interaction-terms, modified impedance matrix, orthogonal projection concept, transmission cost allocation.

I. INTRODUCTION

ONE of the most challenging issues of today's deregulated power industry is to find a fair and efficient Transmission Cost Allocation (TCA) strategy which can recover the total transmission costs, reveal the actual "extent of use" of the transmission system by its users, and provide economic signals for the right placement of new generation and transmission facilities.

The costs associated with the basic transmission services as well as the existing transmission infrastructure, is basically consistent with the fixed transmission cost, also considered as the existing system cost, or the embedded transmission cost [1]. There exist a large number of methodologies in the literature regarding the TCA problem. Conventionally,

the Pro-Rata method was used to allocate the costs among the users [2]. The main concern about the Pro-Rata method is that, it does not consider the actual use of the network, and therefore fails to provide accurate economic signals to the users. Currently, the method is enhanced and utilized to allocate the costs associated with the unused capacity of the transmission facilities [3], [4]. The MW-Mile method, which is now widely applied in the literature [5], [6], was the first solution offered for the TCA problem that considered the actual use of the network by the users.

Topological TCA techniques [7]–[9], trace the power flow in the network using the Proportional Sharing (PS) Principle. The principle was introduced in [7], and confirmed in [8] by the concept of cooperative game theory. The existence as well as the uniqueness of a solution to the tracing problem were also proved in [9]. Marginal and incremental pricing methods [10], [11], utilize the sensitivity indices through the allocation process. As a disadvantage, the results obtained by these methods depend mostly to the slack bus selection. Two concepts are introduced to deal with this issue. One is slack bus independent distribution factors [3], [5], and the other is the optimal distributed slack bus [12], rather than a single slack bus. Nodal pricing approaches use the Locational Marginal Prices (LMPs) to allocate the transmission costs. These methods cannot recover the overall network costs. As a solution, the LMPs are modified either by Ramsey pricing theorem [13] or by certain penalties integrated into the economic dispatch [14]. Transaction-based TCA is first introduced in [15], and extended in [16], [17], where Equivalent Bilateral Exchange (EBE) is formed by the optimization and the flow tracing techniques, respectively. Direct use of optimization approaches to the problem of TCA is an alternative, first developed in [18], that is used in [19] together with the min-max fairness criteria to find the optimal values of the participation coefficients for the generators and loads. Some papers also use the Artificial Intelligence (AI) as an efficient tool for power flow tracing in the network [20].

There are also a group of papers [21]–[23], that utilize the game theory to solve the TCA problem by introducing the cost game notion. The best solution is achieved by either of the Nucleolus [21], the Shapley [22], as well as the Aumann-Shapely Values [23]. Although the method is fair, efficient, and stable, computation time of the algorithm will be considerably high for large test systems. In recent literature, a tendency may be seen towards the cost allocation methods with solid

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economic foundations. In [24], benefit-based transmission cost allocation scheme is proposed. Nevertheless, finding the exact benefit that each transmission facility brings to each market participant is still a challenging issue. Reference [25], presents a new transmission pricing model, which is developed by the flexible mix and match technique. The method builds a general flexible toolbox instead of proposing a new transmission fixed cost allocation method.

A group of papers applied the circuit theory to allocate the costs among the users [23], [26]–[31]. The circuit-theory based approaches may be categorized into three main groups: a) Z-bus model [26], [27] and its modified forms [29], [31], b) Modified Nodal Equation (MNE) model [28], and c) Transformer Analogy (TA) model [30]. The main advantage of the circuit-theory based approaches is that the allocated cost-terms are achieved by the network characteristics. However, it is a challenging issue to find a generally accepted mathematical solution to the problem of identifying the exact contribution of the users into the transmission facilities, because the transmission systems are non-linear and hence, interaction terms exist in equations. Interaction terms are divided between the variables based on different sharing principles, namely, proportional [32], quadratic [33], and equal sharing [23], [31], [34]. A limited number of papers [26]–[30], on the other hand, avoid the interaction terms by taking into account a single variable, at a time. Reference [35], investigates the effects of the criteria used to divide the interaction terms of the loss expressions on power tracing. In [32], it is shown that the allocation results achieved by equal as well as proportional sharing principles are more sensible compared with those of the single variable division. As another limitation, the majority, if not all, the circuit theory based methods provide complex shares for the users rather than real ones, which make it impossible to compare the contributions. Only a limited number of papers devoted their study to this problem. Reference [36], uses the orthogonal projection concept as a basis to allocate the branch losses. In [37], the current projection component is defined to find out the contribution of the users into the network branch currents.

In this paper, a new circuit theory based approach for allocating the transmission costs among the users is proposed. The proposed method is developed by modified Z-bus model, and computes the responsibilities of the generators and loads for the transmission costs, independently. The proposed allocation process uses the equal sharing principle to divide the interaction terms and hence, to identify the mathematical terms corresponding to the contribution of the participants into the branch power flows. It also utilizes a widely accepted physical concept, named, orthogonal projection, to determine the effective contributions of the participants into the network quantities. To confirm that the proposed methodology is reasonable, a rationale is provided for the sharing principle applied, by the concept of cooperative game theory, and also for the orthogonal projection.

The proposed TCA method is validated by the numerical results of different case studies, and outperforms the published ones as it has the following innovative aspects:

1) In the proposed technique the counter flows are less

intense, compared with those of the Z-bus method. This feature is truly valuable, as intense counter-flows lead to cross subsidies, and in turn, bring about unfair competition in power markets.

2) The proposed technique satisfies all conditions desired in a fair and efficient cost allocation method stated before in the text.

3) Although the results obtained by the proposed cost allocation method are less locational compared with those of the tracing based method, the proposed methodology provides tariffs fairly stable against temporal system variations.

4) The contribution of the participants into the sending as well as receiving end flows of the branches are fairly equal under the proposed method, whereas, in case of Z-bus based methods, the results vary significantly when calculation side is changed.

5) Unlike other circuit theory based methods, the proposed technique validates the model applied to treat the non-linear behavior of the transmission networks, and subsequently, to determine the contribution of the participants into the branch power flows.

6) The real shares of the users on the network complex quantities are provided for the first time by the proposed method, which, in turn, make it possible to compare and better assess the contributions.

7) The proposed method is not complicated and computationally demanding even on realistic sized problems, and can perform well in case of large practical power systems, as it overcomes the limitations of the Z-bus as well as the tracing based methods to invert the large-scale sparse matrices, as well.

8) The proposed technique is applicable for all network structures, since the singularity problem with the Z-bus based methods to build the impedance matrix doesn't exist under the proposed method.

II. MODIFIED Z-BUS THEORY

This study presents the cost allocation process for the generators. Thus, the generators and loads are modeled as current injections and equivalent impedances, respectively.

The network actual measured power flow pattern is used to perform the calculation. Note that, if DC Load Flow (DCLF) is applied in the process of a circuit-theory based TCA, it will indeed bring about unrealistic and unfair results that do not reflect the actual transmission use of the users. The reason is that DCLF looks only at the active power and neglects the reactive power that has influence on the active branch power flows through the network, and, subsequently, on transmission system usage. For a power system with N_b buses and N_l lines, let's suppose there are N_g generator buses and N_d demand buses. After a converged load flow is performed, the equivalent currents of the generators will be as follows [26]:

$$I_n = \frac{P_n - jQ_n}{V_n^*} \quad (1)$$

where P_n , Q_n are the active and reactive powers injected at generator bus n , respectively, and V_n is the voltage at bus n .

Similarly, the equivalent admittance of a load at bus d is

$$y_d = \frac{P_d - jQ_d}{|V_d|^2} \quad (2)$$

where P_d , Q_d are the active and reactive demand powers, respectively, and V_d is the voltage at bus d . Note that, considering the loads with their actual models has no effect on the performance of the transmission cost allocation methods, because these methods use the system solved power flow results to compute the contributions.

Equivalent admittances of the loads are involved in the network admittance matrix, and the modified admittance matrix is built. This matrix is then inverted to form the modified impedance matrix, Z_{mod} , which is used to write the relationship between the bus voltages and current injections. The voltage of bus k will then be

$$V_k = \sum_{n \in \Omega_g} z_{kn} \cdot I_n = \sum_{n \in \Omega_g} V_k^n \quad (3)$$

where z_{kn} is the (k, n) element of Z_{mod} , Ω_g is the set of generator buses, and V_k^n is the participation of the generator at bus n in the voltage of bus k .

The current through the branch connecting bus i to bus j is

$$I_{ij} = \frac{1}{2} y_{ij-\text{sh}} \cdot V_i + y_{ij} \cdot (V_i - V_j) \quad (4)$$

where $y_{ij-\text{sh}}$, and y_{ij} are the shunt and series admittances of the branch ij , respectively. Substituting (3) in (4) we have:

$$I_{ij} = \sum_{n \in \Omega_g} \left[\frac{1}{2} y_{ij-\text{sh}} \cdot V_i^n + y_{ij} (V_i^n - V_j^n) \right] = \sum_{n \in \Omega_g} I_{ij}^n \quad (5)$$

where I_{ij}^n is the participation of the generator at bus n in the current through the line ij . Observe that, if based on (3) and (5), V_k^n and I_{ij}^n are used to calculate the share of the generator located at bus n on the voltage of bus k and the current through line ij , respectively; the results would be a complex number. This point is well described in the following section.

Using (3) and (5), we can write the complex power through the line ij

$$\begin{aligned} S_{ij} &= V_i \cdot I_{ij}^* = \left(\sum_{m \in \Omega_g} V_i^m \right) \left(\sum_{n \in \Omega_g} I_{ij}^n \right)^* \\ &= \sum_{n \in \Omega_g} V_i^n (I_{ij}^n)^* + \sum_{\substack{m, n \in \Omega_g \\ m \neq n}} V_i^m (I_{ij}^n)^*. \end{aligned} \quad (6)$$

Note that, due to the electrical power being the product of $V \times I$, multiplication terms, that is, either the self-terms, $V_i^n I_{ij}^n$, as the multiplication of two variables both corresponding to a common current injection or the interaction-terms, $V_i^m I_{ij}^n$, as the multiplication of two variables each corresponding to a distinct current injection, occur in the right hand side of (6).

III. ORTHOGONAL PROJECTION CONCEPT

Let's suppose, A is the sum of two vectors, B and C . The share of B on A is:

$$\frac{B}{A} = \frac{|B| e^{j\theta_B}}{|A| e^{j\theta_A}} \quad (7)$$

where $|A|$, $|B|$ and θ_A , θ_B are the magnitudes and the phase angles of the vectors, A and B , respectively. Observe that, the

result is a complex number. This, in turn, makes it impossible to compare the shares of the involved components, B and C , on A . If, the vectors are resolved into the real and imaginary parts and each part is considered alone, the shares become:

$$\frac{B_x}{A_x} = \frac{|B| \cos \theta_B}{|A| \cos \theta_A}, \quad \frac{B_y}{A_y} = \frac{|B| \sin \theta_B}{|A| \sin \theta_A} \quad (8)$$

where the subscripts x and y denote the real and imaginary parts of a complex number. Although the shares are not complex, the results depend on the calculation reference [37]. As a consequence, this process does not provide a good measure to assess the participations.

To find the real values for the shares of the involved vectors on the resultant one, the notion of vector projection and rejection is used: "The vector projection of B on A is the orthogonal projection of B onto a straight line parallel to A . Likewise, the vector rejection of B on A is the orthogonal projection of B onto a straight line orthogonal to A ". Based on this notion, vectors B and C may be resolved into the components parallel and orthogonal to A , where parallel components sum up to build the vector A , whereas the orthogonal ones are rejected. By definition, the vector projection of B on A is

$$\vec{B}_P = \frac{\vec{B} \cdot \vec{A}}{|\vec{A}|^2} \vec{A} = |\vec{B}| \cos(\theta_B - \theta_A) \vec{a} \quad (9)$$

where B_P is the component of B parallel to A , and a is the unit vector with the same direction as A . Using B_P to calculate the share of B on A , we have:

$$\frac{B_P}{A} = \frac{|B_P| e^{j\theta_A}}{|A| e^{j\theta_A}} = \frac{|B| \cos(\theta_B - \theta_A)}{|A|} \quad (10)$$

Based on (10), the share is a real number and hence, provides a good measure to compare the participations. This solution conforms to a widely accepted physical concept, entitled, the work done by a force. Only the component of the force in the direction of the motion does work. It also makes sense, if applied to the basic electrical quantities named, voltage, current and electric power, considering the fact that, according to the well-known concept of analogous electrical and mechanical systems, voltage and current can be treated as the force applied as well as the movement velocity motivated by the force, where their multiplication results in power in both systems.

IV. SHARING PRINCIPLES OF INTERACTION-TERMS

According to (6), to indicate the branch power flow expression corresponding to each current injection, it is required to divide each interaction term between its involved variables, using a sharing principle. Next, we address the most common sharing principles.

In general, a simple interaction-term division problem may be defined as follows:

$$\begin{cases} f(x_i, x_j) = 2x_i x_j = \alpha_i x_i x_j + \alpha_j x_i x_j \\ \text{s.t. constraint: } \alpha_i + \alpha_j = 2 \end{cases} \quad (11)$$

where x_i and x_j are two variables involved in the interaction term, $2x_i x_j$, and α_i , α_j are the participation coefficients

of the variables, respectively. To divide the interaction term between the involved variables, it is necessary to calculate the participation coefficients. Three common sharing principles are used to calculate the coefficients, namely, proportional, quadratic, and equal or 50-50 sharing.

According to the proportional sharing principle, the coefficients α_i, α_j , are related directly with their relevant variables. Therefore,

$$\frac{\alpha_i}{x_i} = \frac{\alpha_j}{x_j}. \quad (12)$$

Using (12) in equality constraint of (11), results in

$$\alpha_i = \frac{x_i}{x_i + x_j}, \alpha_j = \frac{x_j}{x_i + x_j}. \quad (13)$$

The coefficients α_i, α_j , are, however, related with the square of their relevant variables under the quadratic sharing principle,

$$\frac{\alpha_i}{x_i^2} = \frac{\alpha_j}{x_j^2}. \quad (14)$$

Given the constraint of (11), we have

$$\alpha_i = \frac{x_i^2}{x_i^2 + x_j^2}, \alpha_j = \frac{x_j^2}{x_i^2 + x_j^2}. \quad (15)$$

In case of equal sharing principle, the coefficients α_i, α_j are assigned equal and unitary values:

$$\alpha_i = \alpha_j = 1 \quad (16)$$

Since the sharing principles cannot be mathematically proved, the rationale behind any sharing principle must be confirmed. Similar cases arise in economics, where a joint cost is allocated between the cost objects in a cooperative way. The cooperative game theory is suitable for this concept, because its main focus is to faithfully allocate the joint costs (or benefits) among the parties. In this paper, we attempt to provide a rationale for the equal sharing principle applied to divide the interaction terms. In this way, we consider the interaction term sharing problem stated in (11) as a cooperative joint cost game formulation. The formulated cost game is a hypothetical game, which is used to confirm the rationale behind the equal sharing principle. Accordingly, it is revealed that in an interaction term division problem, the equal sharing principle brings about results that follow exactly the solutions achieved by the Shapley as well as Aumann-Shapley values.

In the subsequent section, the notion of cooperative game theory and its solutions are more explained.

V. PREFERRED COST GAME SOLUTIONS

A cost game is defined with the pair $(\mathbf{N}, c)$, where $\mathbf{N} = [1, 2, \dots, n]$ is the set of agents and c is a function that specifies the cost of each coalition (subset) of $\mathbf{N}$. Any cost allocation is a game solution given by a vector $x \in R^n$, where the allocated cost to agent i is denoted by element x_i of vector x . If a solution exists from which all of the agents profited, compared with all other possible coalitions, it can be considered as the game balance, and is also said to be

in the core of the game. For a given solution, the following rationality conditions are required to be in the core:

$$\text{Global rationality } \sum_{i \in \mathbf{N}} x_i = c(\mathbf{N}) \quad (17a)$$

$$\text{Group/individual rationality } \sum_{i \in S} x_i \leq c(S) \text{ for all } S \subseteq \mathbf{N} \quad (17b)$$

For the cost game defined, the Shapley value as a single point game theory solution, assigns the cost of agent i , denoted by $\Phi_i (i = 1, 2, \dots, n)$, as follows:

$$\Phi_i = \sum_{S \subseteq \mathbf{N} \setminus \{i\}} \frac{|S|! (n - |S| - 1)!}{n!} [v(S \cup \{i\}) - v(S)] \quad (18)$$

where $v(\cdot)$ is the gain function, S is any coalition with the i^{th} agent excluded, $|S|$ is the number of agents in coalition S . The Shapley value solution doesn't necessarily meets the rationality condition of (17b). As a result, it may not belong to the core of the game. However, it is shown in [38] that in the context of whole network game, the group rationality often holds true for the Shapley value, and subsequently, the solution belongs to the core as well. Reference [39], in addition, states that the Shapley value solution belonging to the core, brings about more desirable properties than other single point solutions.

The Aumann-Shapley is another single point solution which is derived directly from the Shapley value. The idea is to divide each agent into infinitesimal sub-agents, and then apply the Shapley value to each one taking each sub-agent as an individual [23].

Although the game-theoretic solutions are fair and efficient, implementing the game theory to allocate the costs among the participants becomes a relatively complicated and computationally demanding task, when the system under study is sufficiently large. However, these solutions might be used to assess any other usage-based cost allocation method in terms of fairness and efficiency, and also to confirm whether a mathematical model is fair and reasonable.

VI. PROPOSED METHOD

In this section, first, the rationale behind the equal sharing of interaction terms is confirmed by the concept of game theory. Subsequently, the equal sharing principle is used to divide the interaction terms of the branch power flow equations between their involved variables. The contributions of each participant into the branch power flows are then determined. After that, the orthogonal projection notion is used to introduce the concept of effective contributions as well as the real shares of the users on the network quantities. The effective contributions of the users into the branch power flows are also derived directly from their effective contributions into the bus voltages as well as the branch currents. Finally, the network costs are allocated to the users based on their contributions into the branch power flows. Fig. 1 depicts the flowchart of the proposed cost allocation method.

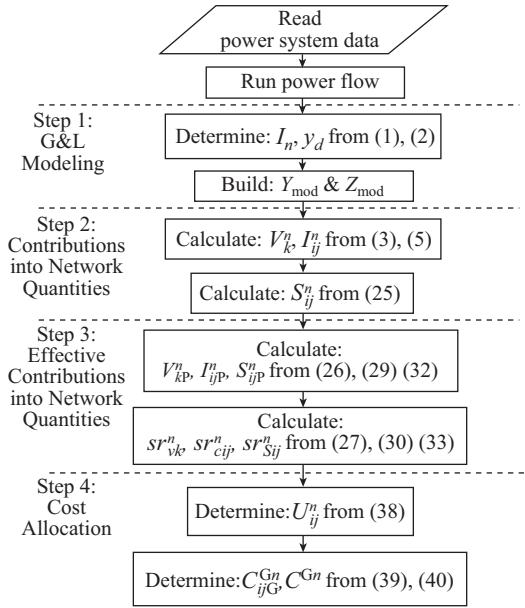


Fig. 1. Flowchart of the proposed cost allocation method.

A. Validating the Equal Sharing Principle

Given $f(x_i, x_j) = 2x_i x_j$ of (11) as the joint cost of two participants x_i and x_j , we define a cost game with two players x_i and x_j and the cost function c equal to f . The objective is to divide up the joint cost between the participants in an efficient and acceptable way.

As stated previously, the Shapley and the Aumann-Shapley values are suitable solution concepts because they have all the appropriate properties required for a cost allocation scheme. The agents are satisfied with the allocation results, because the solutions are in the core of the game.

Next, the Shapley value is utilized to solve the general interaction term division problem, defined by (11).

The problem is described as follows: how to assign $2x_i x_j$ to each of x_i , and x_j . The gains $v(\cdot)$, are to be computed first, for all possible coalitions of two variables, x_i , and x_j :

$$v(\{i\}) = 2x_i \times 0 = 0; v(\{j\}) = 0; v(\{i, j\}) = 2x_i x_j. \quad (19)$$

Then, (18) is used to calculate the imputation component of $f(x_i, x_j)$ to source x_i , denoted by f^{x_i} :

$$f^{x_i} = \frac{0!(2-0-1)!}{2} [v(\{i\}) - 0] + \frac{1!(2-1-1)!}{2} [v(\{i, j\}) - v(\{j\})] = \frac{1}{2} 2x_i x_j = x_i x_j. \quad (20)$$

Again, (18) is used to calculate the imputation component of $f(x_i, x_j)$ to source x_j . The results confirm the equal sharing of interaction terms, because the shares of x_i and x_j on $2x_i x_j$, are the same and each equals $x_i x_j$.

We also use the Aumann-Shapley value to divide the interaction term, $f(x_i, x_j)$, between its involved components, x_i, x_j . According to the solution concept of Aumann-Shapley value, Unitary Participation (UP) of x_i into the $f(x_i, x_j)$ is:

$$UP_{(x_i \rightarrow f)} = \int_{t=0}^1 \frac{\partial f(tx)}{\partial x_i} dt = \int_{t=0}^1 2(tx_j) dt = t^2 x_j \Big|_0^1 = x_j \quad (21)$$

where each agent is divided into infinitesimal parts, $(\Delta x \rightarrow 0)$, by infinitesimal steps, t .

Likewise, the UP of x_j into the $f(x_i, x_j)$ is:

$$UP_{(x_j \rightarrow f)} = \int_{t=0}^1 \frac{\partial f(tx)}{\partial x_j} dt = \int_{t=0}^1 2(tx_i) dt = t^2 x_i \Big|_0^1 = x_i. \quad (22)$$

The Total Participation (TP) of the agents into the $f(x_i, x_j)$, is achieved by multiplying the UP with the amount of the agent:

$$\begin{cases} TP_{(x_i \rightarrow f)} = UP_{(x_i \rightarrow f)} \times x_i = x_i x_i \\ TP_{(x_j \rightarrow f)} = UP_{(x_j \rightarrow f)} \times x_j = x_i x_j \end{cases} \quad (23)$$

The outcomes absolutely confirm the equal sharing of interaction terms.

B. Branch Power Flow Contribution of the Generators

Take the branch power flow through line ij , expressed by (6). The problem is to specify the partial branch power flows through line ij , corresponding to the current injections of the generators. The equal sharing principle is used to divide the interaction terms. Therefore, the allocation process assigns each individual-term entirely to its corresponding generator, and, also assigns half of each interaction-term to its corresponding generators, involved. By definition, the contribution of the generator at bus n into the power flow through the branch ij will be

$$S_{ij}^n = V_i^n (I_{ij}^n)^* + \frac{1}{2} V_i^n \sum_{\substack{m \in \Omega_g \\ m \neq n}} (I_{ij}^m)^* + \frac{1}{2} (I_{ij}^n)^* \sum_{\substack{m \in \Omega_g \\ m \neq n}} V_i^m \quad (24)$$

where, on the right hand side of the equation, first term is the individual contribution term, whereas second and third terms are the common contribution terms. Incorporating the individual term into the interaction terms and substituting the equivalents using (3) and (5), we obtain

$$\begin{aligned} S_{ij}^n &= \frac{1}{2} [V_i^n \sum_{m \in \Omega_g} (I_{ij}^m)^* + (I_{ij}^n)^* \sum_{m \in \Omega_g} V_i^m] \\ &= \frac{1}{2} (V_i^n I_{ij}^{*n} + V_i^n I_{ij}^n). \end{aligned} \quad (25)$$

Observe that, if S_{ij}^n is used directly to compute the share of the generator at bus n on the power flow through the line ij , the result will become a complex number. Next, we define the concept of effective contributions and then assign real values to the shares of the generators on the network quantities. This, in turn, makes it possible to compare the contributions.

C. Effective Contributions of the Generators into the Network Quantities

Given that V_k^n denotes the participation of the generator at bus n in the voltage of bus k , based on (9), the orthogonal projection of V_k^n on V_k will become

$$\vec{V}_{kP}^n = \frac{\vec{V}_k^n \cdot \vec{V}_k}{|\vec{V}_k|^2} \vec{V}_k = |\vec{V}_k^n| \cos \delta_{kn} \frac{\vec{V}_k}{|\vec{V}_k|} \quad (26)$$

where $\vec{V}_k^n \cdot \vec{V}_k$ and δ_{kn} are the dot product and phase angle between $\vec{V}_k^n$ and $\vec{V}_k$, respectively. V_{kP}^n is defined as the

effective contribution of the generator at bus n , into the voltage at bus k . We use V_{kP}^n to calculate the share of the generator at bus n on the voltage of bus k ,

$$sr_{vk}^n = \frac{V_{kP}^n}{V_k} = \frac{|V_{kP}^n| e^{i\theta_k}}{|V_k| e^{i\theta_k}} = \frac{|V_{kP}^n|}{|V_k|} \cos \delta_{kn} \quad (27)$$

where θ_k is the phase angle of the voltage at bus k . It is observed that, the share is a real number. Subsequently, the percentage share is provided which represents part of a whole.

V_k can now be written as the algebraic sum of the effective contributions of the generators,

$$V_k = \sum_{n \in \Omega_g} V_{kP}^n = \sum_{n \in \Omega_g} |V_{kP}^n| e^{i\theta_k}. \quad (28)$$

Likewise, the effective contribution of the generator at bus n into the current through line ij is a vector parallel to I_{ij} , built by projecting I_{ij}^n on I_{ij} ,

$$\vec{I}_{ijP}^n = \frac{\vec{I}_{ij}^n \cdot \vec{I}_{ij}}{|\vec{I}_{ij}|^2} \vec{I}_{ij} = |\vec{I}_{ij}^n| \cos \delta_{cij,n} \frac{\vec{I}_{ij}}{|\vec{I}_{ij}|} \quad (29)$$

where $\vec{I}_{ij}^n \cdot \vec{I}_{ij}$, $\delta_{cij,n}$ are the dot product and the phase angle between $\vec{I}_{ij}^n$ and $\vec{I}_{ij}$, respectively. We use sr_{cij}^n to denote the share of the generator at bus n on I_{ij} ,

$$sr_{cij}^n = \frac{I_{ijP}^n}{I_{ij}} = \frac{|I_{ijP}^n| e^{i\theta_{cij}}}{|I_{ij}| e^{i\theta_{cij}}} = \frac{|I_{ijP}^n|}{|I_{ij}|} \cos \delta_{cij,n} \quad (30)$$

where θ_{cij} is the phase angle of the branch current I_{ij} . I_{ij} can now be written as the algebraic sum of the effective contributions of the generators, as follows:

$$I_{ij} = \sum_{n \in \Omega_g} I_{ijP}^n = \sum_{n \in \Omega_g} |I_{ijP}^n| e^{i\theta_{cij}}. \quad (31)$$

We use the same principle to find the effective contribution of the generator at bus n into the power flow of the branch ij ,

$$\vec{S}_{ijP}^n = \frac{\vec{S}_{ij}^n \cdot \vec{S}_{ij}}{|\vec{S}_{ij}|^2} \vec{S}_{ij} = |\vec{S}_{ij}^n| \cos \delta_{Sij,n} \frac{\vec{S}_{ij}}{|\vec{S}_{ij}|} \quad (32)$$

where $\vec{S}_{ij}^n \cdot \vec{S}_{ij}$, $\delta_{Sij,n}$ are the dot product and the phase angle between $\vec{S}_{ij}^n$ and $\vec{S}_{ij}$, respectively. $\vec{S}_{ijP}^n$, is the component of $\vec{S}_{ij}^n$ parallel to $\vec{S}_{ij}$, and is defined as the effective contribution of the generator at bus n into the power flow of the branch ij . Similarly, the share of the generator at bus n on S_{ij} , denoted by sr_{Sij}^n , will be

$$sr_{Sij}^n = \frac{S_{ijP}^n}{S_{ij}} = \frac{|S_{ijP}^n| e^{i\theta_{Sij}}}{|S_{ij}| e^{i\theta_{Sij}}} = \frac{|S_{ijP}^n|}{|S_{ij}|} \cos \delta_{Sij,n} \quad (33)$$

where θ_{Sij} is the phase angle of the power flow through branch ij . S_{ij} may now be written as the algebraic sum of the effective contributions of the generators:

$$S_{ij} = \sum_{n \in \Omega_g} S_{ijP}^n = \sum_{n \in \Omega_g} |S_{ijP}^n| e^{i\theta_{Sij}} \quad (34)$$

D. Effective Contribution of the Generators into the Branch Power Flows: Direct Derivation

If we rewrite (6) and substitute the bus voltage and the branch current by their equivalents of (28) and (31), respectively, the power flow through the line ij will become

$$\begin{aligned} S_{ij} &= V_i I_{ij}^* = \left(\sum_{n \in \Omega_g} V_{iP}^n \right) \left(\sum_{n \in \Omega_g} I_{ijP}^n \right)^* \\ &= \sum_{n \in \Omega_g} [V_{iP}^n I_{ijP}^{n*} + \frac{1}{2} V_{iP}^n \sum_{\substack{m \in \Omega_g \\ m \neq n}} I_{ijP}^{m*} + \frac{1}{2} I_{ijP}^{n*} \sum_{\substack{m \in \Omega_g \\ m \neq n}} V_{iP}^m] \\ &= \sum_{n \in \Omega_g} S_{ijPP}^n \end{aligned} \quad (35)$$

where S_{ijPP}^n is the effective contribution of generator at bus n into the power flow of line ij , derived directly by the bus voltage as well as the branch current. Note that, the equal sharing principle is again used to divide the interaction terms of S_{ij} between the involved variables.

If (34) is compared with (35), and (35) is more simplified, then

$$S_{ijP}^n = S_{ijPP}^n = \frac{1}{2} [V_{iP}^n I_{ij}^* + V_i I_{ijP}^{n*}]. \quad (36)$$

If we use (27) and (30) to substitute the equivalents of V_{iP}^n and I_{ijP}^n in (36), respectively, the share of the generator at bus n on the power flow through line ij , will become

$$sr_{Sij}^n = \frac{S_{ijP}^n}{S_{ij}} = \frac{\frac{1}{2} [V_{iP}^n I_{ij}^* + V_i I_{ijP}^{n*}]}{V_i I_{ij}^*} = \frac{1}{2} [sr_{vi}^n + sr_{cij}^n] \quad (37)$$

According to (37), it is observed that the share of the generator at bus n on the power flow of line ij , equals the average shares of that generator on the sending-end bus voltage as well as the current flow of that line.

E. Transmission Cost Allocation

Unlike a few number of papers [30], for example, in which the transmission cost is allocated based on the partial currents rather than power flows, the proposed method, as with majority of the published papers [23], [26]–[29], [31]–[34], uses the power flows to determine the line usage of the participants. Furthermore, Zero Counter Flow (ZCF) pricing scheme is used to allocate the line costs among the generators and the loads. Thus, users neither pay money nor take a reward for usage of the lines for which their power flow contributions are in opposite direction. Accordingly, we define the usage of line ij due to the generator at bus n , as

$$U_{ij}^n = \begin{cases} S_{ijP}^n & \text{if } S_{ijP}^n \text{ is direct flow} \\ 0 & \text{if } S_{ijP}^n \text{ is counter flow} \end{cases} \quad (38)$$

For each branch, an overall annualized branch cost in \$/h, C_{ij} , is considered. This total cost includes the costs of maintenance, depreciation, and return on capital invested, known as infrastructure costs, and the costs of reserve, congestion management, black-start, voltage control reactive power, system balancing, and losses, known as system service costs. The line cost, C_{ij} , is apportioned to the generators and loads,

denoted by C_{ijG} and C_{ijL} , respectively. In this way, the cost of line ij allocated to the generator at bus n is

$$C_{ijG}^{Gn} = \frac{U_{ij}^n}{\sum_{n \in \Omega_g} U_{ij}^n} C_{ijG}. \quad (39)$$

Finally, the entire transmission network cost allocated to the generator located at bus n is

$$C^{Gn} = \sum_{(i,j) \in \Omega_L} C_{ijG}^{Gn} \quad (40)$$

where Ω_L , is the set of all transmission lines.

VII. NUMERICAL RESULTS

A. A Sample 2-bus System

To explain the implementation of the proposed approach, consider the sample two-bus system displayed in Fig. 2, with two generators connected to bus 1 and a load at bus 2. The line impedance is $(0.1 + j0.2)$ p.u., and the per unit values of the network quantities are depicted in Table I, calculated on a base of 100 MVA and 100 kV.

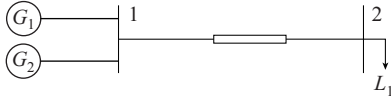


Fig. 2. Sample 2-bus system.

Integrating Y_{L1} into the network admittance matrix, Y_{mod} and Z_{mod} will be

$$Y_{mod} = \begin{pmatrix} 2 - j4 & -2 + j4 \\ -2 + j4 & 3.70 - j3.84 \end{pmatrix},$$

$$Z_{mod} = \begin{pmatrix} 0.68 + j0.15 & 0.58 - j0.05 \\ 0.58 - j0.05 & 0.58 - j0.05 \end{pmatrix} \quad (41)$$

Now, the proposed method is applied to find the share of the generators on the bus voltages, the branch current as well as the power flows. The calculation results are depicted in Table

II. For example, to find the share of G1 on V_1 , we use (3), (26), and (27) consecutively,

$$V_1^{G1} = Z_{11} I_{G1} = 0.36 \angle 10.78^\circ$$

$$V_{1P}^{G1} = |V_1^{G1}| \cos \delta_{11} \angle \theta_1 = 0.36 \angle 10^\circ$$

$$sr_{V1}^{G1} = \frac{V_{1P}^{G1}}{V_1} = \frac{0.36 \angle 10^\circ}{1 \angle 10^\circ} = 0.36 \quad (42)$$

Similarly, (5), (29) and (30) are used to find the share of G1 on I_{12} . Finally, we may use either (32) or (36) to calculate the effective contribution of G1 on S_{12} :

$$S_{12P}^{G1} = |S_{12}^{G1}| \cos \delta_{S12,G1} \angle \theta_{S12} = 0.51 \angle 0.95^\circ$$

$$S_{12PP}^{G1} = \frac{1}{2} [V_1 (I_{12P}^{G1})^* + V_{1P}^{G1} (I_{12})^*] = 0.51 \angle 0.95^\circ \quad (43)$$

Note, first of all, that the allocation cannot be performed using Z-bus method, because the network Y-bus becomes singular as a consequence of shunt capacitance of the branch being ignored. However, the proposed method overcomes the singularity problem of building Z-bus matrix of the sample 2-bus system by integrating the equivalent impedance of the load L1 to the Y-bus matrix. In addition, in the context of Z-bus method, even small changes of network parameters affect the values of Z-bus, specifically at the sub-transmission levels where shunt capacitance is small.

Table II reveals that the proposed method provides percentage shares of G1 and G2 on the network quantities including the voltage of buses 1 and 2, the current and power flow of the branch 1-2, thanks to the consideration of the effective contributions. This, in turn, helps to better assess the contributions by comparing the results. A careful analysis of the results confirms that the share of G2 on each of the network quantities is correctly more than that of G1 by the proposed method, because G2 feeds almost 64% of the load L1.

Equation (42) also confirms that, as proved in the previous section, both S_{12P}^{G1} and S_{12PP}^{G1} bring about the same results.

TABLE I
PER UNIT VALUES OF THE SAMPLE 2-BUS SYSTEM QUANTITIES

	Bus Number		Line Number
	1	2	1-2
Voltage	$V_1 = 1 \angle 10^\circ$	$V_2 = 0.84 \angle -7.36^\circ$	$\Delta V_{1-2} = 0.32 \angle 61.34^\circ$
Power	$S_{G1} = 0.5 + j0.1$	$S_{L1} = 1.195 - j0.11$	$S_{12} = 1.432 \angle 12.095^\circ$
	$S_{G2} = 0.9 + j0.2$		
Current/ Admittance	$I_{G1} = 0.509 \angle -1.31^\circ$	$Y_{L1} = 1.708 \angle 5.26^\circ$	$I_{12} = 1.432 \angle -2.095^\circ$
	$I_{G2} = 0.922 \angle -2.53^\circ$		

TABLE II
THE SHARE OF THE GENERATORS ON THE SAMPLE 2-BUS SYSTEM QUANTITIES BY THE PROPOSED METHOD

Generator Number						
G1		G2				
	Contributions (p.u.)	Effective contributions (p.u.)	Share (%)	Contributions (p.u.)	Effective contributions (p.u.)	Share (%)
V_1	$V_1^{G1} = 0.36 \angle 10.78^\circ$	$V_{1P}^{G1} = 0.36 \angle 10^\circ$	36	$V_1^{G2} = 0.64 \angle 9.56^\circ$	$V_{1P}^{G2} = 0.64 \angle 10^\circ$	64
V_2	$V_2^{G1} = 0.30 \angle -6.57^\circ$	$V_{2P}^{G1} = 0.3 \angle -7.36^\circ$	35.7	$V_2^{G2} = 0.54 \angle -7.79^\circ$	$V_{2P}^{G2} = 0.54 \angle -7.36^\circ$	64.3
I_{12}	$I_{12}^{G1} = 0.51 \angle -1.34^\circ$	$I_{12P}^{G1} = 0.51 \angle -2.095^\circ$	35.7	$I_{12}^{G2} = 0.92 \angle -2.53^\circ$	$I_{12P}^{G2} = 0.92 \angle -2.095^\circ$	64.3
S_{12}	$S_{12}^{G1} = 0.51 \angle 12.11^\circ$	$S_{12P}^{G1} = 0.51 \angle 12.095^\circ$	35.8	$S_{12}^{G2} = 0.92 \angle 12.08^\circ$	$S_{12P}^{G2} = 0.92 \angle 12.095^\circ$	64.2

B. IEEE 30-bus Test System

The IEEE 30-bus system, illustrated in Fig. 3, is used to validate the proposed method. Branch data are provided in [40]. Table III, shown in appendix, provides bus data for the base case. The proposed method is implemented on the test system, along with two well-known transmission cost allocation techniques, namely, Z-bus [27], and PS method [7],

and the results are compared. For each line, the annualized line cost in \$/h, is considered 1000 times its series reactance. The proposed and PS methods, allocate the network costs between the generators and loads by 50-50 ratio.

To compare the three methods, the branch real power flows are considered. Fig. 4 illustrates the share of the generators on the real power flow of the branches for the three methods.

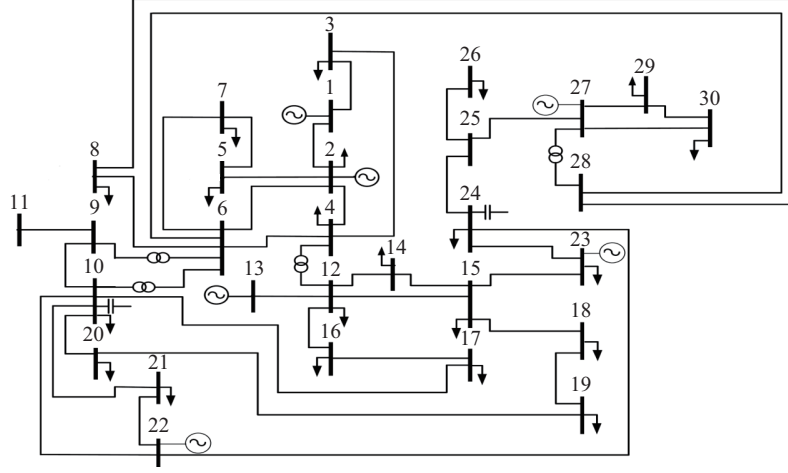


Fig. 3. IEEE 30-bus test system.

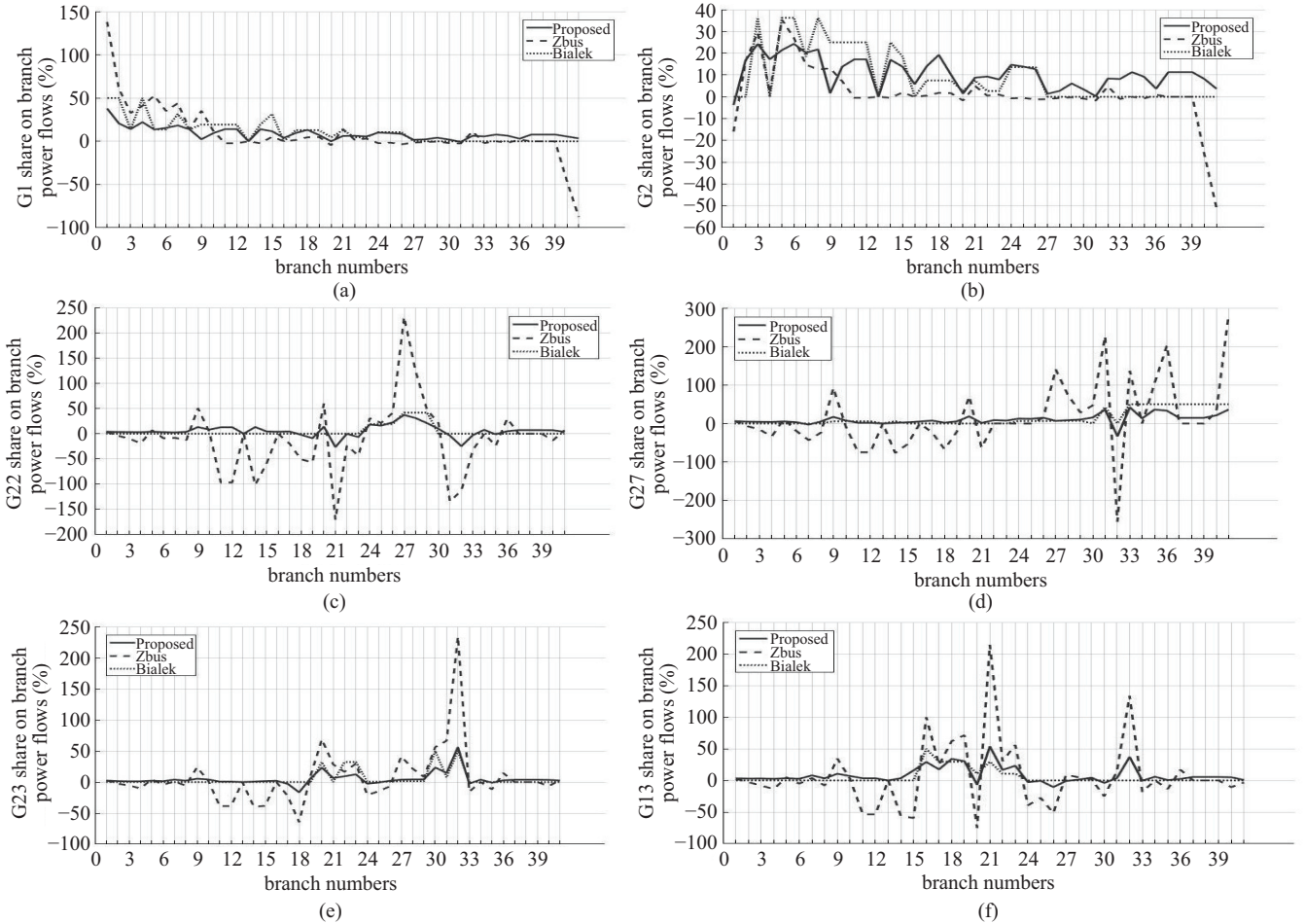


Fig. 4. Branch real power flow shares of the generators. (a) G1. (b) G2. (c) G22. (d) G27. (e) G23. (f) G13.

TABLE III
LOAD FLOW RESULTS FOR IEEE 30-BUS TEST SYSTEM

Bus	Voltage		Generation		Load	
	Magnitude (p.u.)	Angle (degree)	Real (MW)	Reactive (MVAR)	Real (MW)	Reactive (MVAR)
1	0.9824	0	41.54	-5.44	0	
2	0.9787	-0.7630	55.40	1.67	21.7	12.7
3	0.9769	-2.3897	0	0	2.4	1.2
4	0.9764	-2.8386	0	0	7.6	1.6
5	0.9713	-2.4864	0	0	0	0
6	0.9723	-3.2287	0	0	0	0
7	0.9623	-3.4910	0	0	22.8	10.9
8	0.9611	-3.6819	0	0	30	30
9	0.9903	-4.1371	0	0	0	0
10	0.9998	-4.5998	0	0	5.8	2
11	0.9903	-4.1371	0	0	0	0
12	1.0174	-4.4979	0	0	11.2	7.5
13	1.0645	-3.2980	16.20	35.93	0	0
14	1.0066	-5.0397	0	0	6.2	1.6
15	1.0092	-4.8140	0	0	8.2	2.5
16	1.0028	-4.8393	0	0	3.5	1.8
17	0.9955	-4.8873	0	0	9	5.8
18	0.9933	-5.4843	0	0	3.2	0.9
19	0.9873	-5.6882	0	0	9.5	3.4
20	0.9896	-5.4719	0	0	2.2	0.7
21	1.0093	-4.6208	0	0	17.5	11.2
22	1.0160	-4.5030	22.74	34.19	0	0
23	1.0256	-3.7557	16.27	6.96	3.2	1.6
24	1.0167	-3.8852	0	0	8.7	6.7
25	1.0438	-2.0724	0	0	0	0
26	1.0267	-2.4760	0	0	3.5	2.3
27	1.0690	-0.7147	39.91	31.75	0	0
28	0.9820	-3.2152	0	0	0	0
29	1.0500	-1.8494	0	0	2.4	0.9
30	1.0391	-2.6429	0	0	10.6	1.9

According to the profiles, all the three methods fairly reflect the locational signals, that is, the generators have large shares on the direct flow of the neighboring branches. For the proposed method, generator G22, for example, contributes most in direct flows of the lines 24–32 (Fig. 4(c)). The dominant shares of

generators G27, G23, and G13 on branch direct flows, under proposed method, correspond to the lines (30–41), (20–23, 30–32), and (15–23, 32), respectively. For G1, and G2, the share profiles are less locational, which imply that these generators, located at the center of the network with largest output powers, feed different loads in the network.

In addition, the shares of the generators in Fig. 4 are studied in terms of counter flows. It is shown that the proposed and Z-bus methods, both take into account the counter-flows, whereas counter-flows are disregarded by the PS method. This property is well reflected on the corresponding profiles of PS method by a relatively large number of points with zero values. Furthermore, it can be seen from the profiles of the Z-bus method that the dominant shares are more intense, both on direct as well as counter flows. In the proposed technique, however, the counter flows are less intense, compared with those of the Z-bus method. This feature is truly valuable, as intense counter-flows lead to cross subsidies, bring about unfair competition in power markets, and also diverse results for different MW-Mile pricing schemes.

Figure 5 depicts the allocation of the transmission lines' costs among the generators, under each of the three methods. The dotted lines represent the branch costs and bar graphs demonstrate the allocated branch costs to the generators. As shown in Fig. 5, the proposed and PS methods, both allocate the cost of the lines by 50-50 ratio between the generators and loads. On the other hand, by the Z-bus method, the cost allocation ratio between the generators and the loads is determined based on the network parameters. According to the Fig. 5(a), the proposed method exhibits the locational characteristics, i.e., the cost of a line is allocated mostly to its nearby generators. This feature is most prominent by the PS method (Fig. 5(c)). Given branch 1 with branch cost 60 \$/h, connected between buses 1 and 2, for example, the

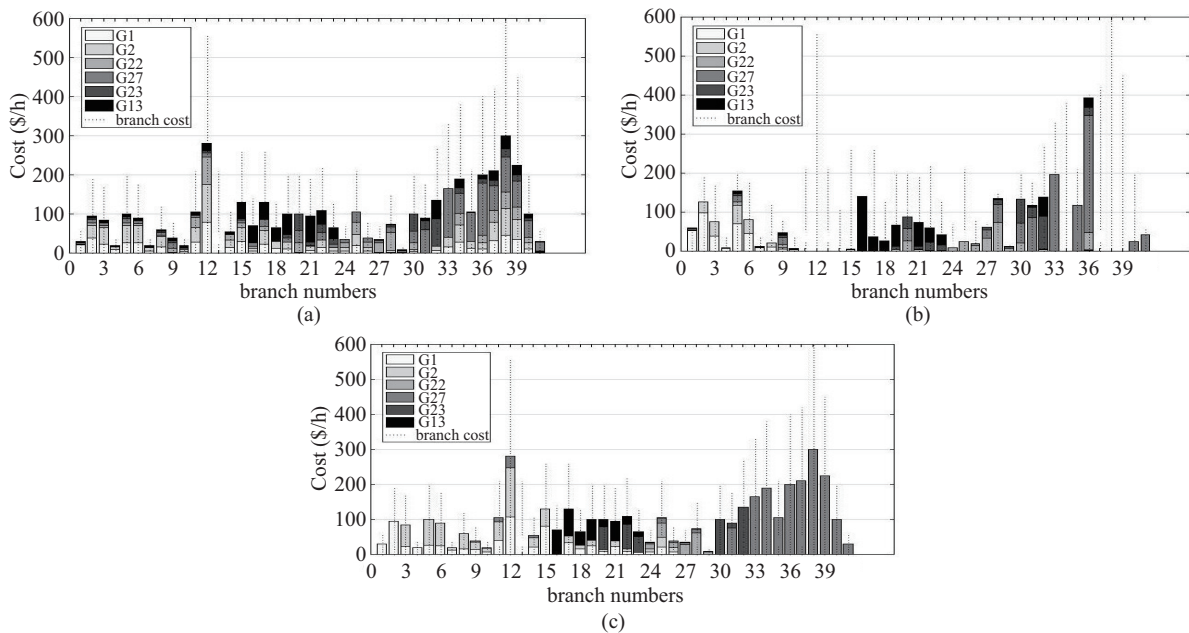


Fig. 5. The cost allocation of the branches between the generators. (a) Proposed method. (b) Z-bus method. (c) PS method.

contribution of G1 is dominant, under the three methods, since it is located at the sending end bus of branch 1. As the pricing scheme ignores the counter-flows, the share of G2 on power flow of branch 1 is zero. The proposed technique (Fig. 5(a)), assigns 30 \$/h (50%) of branch 1 cost to the set of generators with 21.1494 \$/h, 0 \$/h, 2.2644 \$/h, 3.3851 \$/h, 1.3690 \$/h, and 1.8321 \$/h allocated costs to G1, G2, G22, G27, G23, and G13, respectively, and 30 \$/h (50%) of line 1 cost to the set of loads. The PS method (Fig. 5(c)), as with the proposed method, assigns 30 \$/h (50%) of branch 1 cost to the set of generators with 30 \$/h allocated cost to G1, and also assigns 30 \$/h (50%) of line 1 cost to the set of loads. Under the Z-bus method (Fig. 5(b)), however, 59.7805 \$/h of branch 1 cost is assigned to the set of generators with 53.6783 \$/h, 0 \$/h, 1.4544 \$/h, 2.6048 \$/h, 1.0270 \$/h, and 1.0160 \$/h allocated costs to G1, G2, G22, G27, G23, and G13, respectively, and 0.2195 of line 1 cost to the set of loads. The same logic holds for the rest of the branches.

Figure 6 shows the Transmission Use of System (TUoS) costs of the generators. To better indicate the locational signals provided by the methods, the per-unit costs of the generators are also depicted in Fig. 6. Note from the TUoS cost allocation profiles that, the costs allocated to the generators G1 and G2, for the Z-bus method are small, in spite of their highest output powers, because under the Z-bus method they only have shares on the costs of less expensive branches, as shown in Fig. 5(b). This in turn yields to a low set of TUoS tariffs for the G1 and G2. For the proposed method, the amount of TUoS costs allocated to the G23, G13 and G22 with lower output powers are less compared to those of the generators G1, G2, and G27. According to the TUoS charges of the generators (Fig. 6), it can be concluded that each of the three methods, signify G27, G23, and G13 as the ones with high TUoS tariffs; whereas, they suggest G1 and G2 with lower TUoS tariffs, because big loads are located at the buses (2, 7,

8, 21). Relatively high discrepancies arise for TUoS charge of G22 and G27, calculated by the methods. The tariffs calculated by the PS method, reflect the relative distances between the generators and the loads; whereas, under the proposed method, they reflect the relative electrical conditions of the generators established by the network voltage and current equations.

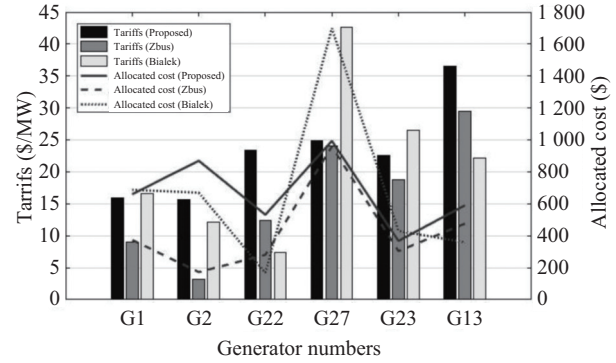


Fig. 6. TUoS costs and charges of the generators.

Furthermore, the proposed method is compared with Z-bus method in terms of sensitivity to the calculation reference side of the branches. The results are illustrated in Table IV. According to the results, under the Z-bus method, the TUoS costs and charges of G1 and G2 noticeably change when the calculation side is altered; whereas for the proposed method, small changes are observed.

The proposed method is also evaluated in terms of the sensitivity to the load variations. Table V depicts the transmission per-unit costs assigned to the generators for multiple load scenarios each with a single load varied, compared to the based case. The generators G1 and G2 tariffs are extremely sensitive to the variations of the large loads located at buses 2, 7 and 8. Therefore, it is expected that the G1 and G2 tariffs remain

TABLE IV
SENSITIVITY OF THE RESULTS TO THE CALCULATION REFERENCE SIDE OF THE IEEE 30-BUS BRANCHES

Gen. buses	Tariffs (\$/MWh)				Costs (\$/h)			
	Proposed		Z-bus		Proposed		Z-bus	
	Sending end	Receiving end	Sending end	Receiving end	Sending end	Receiving end	Sending end	Receiving end
G1	15.8951	15.8551	9.01	4.4232	660.3	658.7	374.3	183.7
G2	15.7000	15.6326	3.1239	1.4017	869.8	866.1	173.1	77.6
G22	23.3994	23.4433	12.3522	12.9168	532.1	533.1	280.9	293.7
G27	24.9016	25.0856	24.0326	24.9129	993.8	1001.1	959.1	994.2
G23	22.6165	22.5792	18.7133	19.0427	367.9	367.3	304.4	309.8
G13	36.4853	36.3408	29.4215	29.9407	591.1	588.7	476.6	485.1
Total	-	-	-	-	4015	4015	2568.4	2344.1

TABLE V
SENSITIVITY OF THE RESULTS TO THE LOAD VARIATIONS

Gen. bus	Tariffs (\$/MWh)				Z-bus method				PS method			
	Proposed method				Base*				Base*			
	Base*	$P_L(7) = 0$	$P_L(21) = 0$	$P_L(30) = 0$	Base*	$P_L(7) = 0$	$P_L(21) = 0$	$P_L(30) = 0$	Base*	$P_L(7) = 0$	$P_L(21) = 0$	$P_L(30) = 0$
G1	15.9	16.8	15.3	16.5	9.0	10.9	9.7	9.3	16.6	19.8	16.8	16.7
G2	15.7	16.6	15.1	16.5	3.1	3.3	3.2	3.2	12.1	16.3	12.3	12.1
G22	23.4	24.8	33.7	25.9	12.3	12.6	18.2	13.1	7.4	7.2	16.4	7.3
G27	24.9	27.2	26.6	26.8	24.0	24.9	30.9	27.2	42.6	43.8	50.3	53.9
G23	22.6	28.4	21.8	22.9	18.7	18.8	13.7	19.8	26.5	27.8	20.5	26.9
G13	36.5	51.9	40.4	40.2	29.4	35.6	27.4	31.0	22.2	23.8	21.5	22.4

*Base: $P_L(7) = 22.8$ MW, $P_L(21) = 17.5$ MW, $P_L(30) = 10.6$

almost unchanged, if the loads located at buses relatively far from G1 and G2, are altered. This fact is shown by changing the loads located at buses (7, 21, 30). According to the results, the circuit theory based methods have higher tariff stability against the load variations, compared with PS method.

As can be seen in Fig. 6, and Table IV, under the proposed method, the costs allocated to the generators are higher than those of the Z-bus method. This is because under the proposed method, the generators are responsible for half of the overall network cost. Under the Z-bus method, however, since the generators are treated the same way as loads, both as nodal currents, the network cost responsibility ratio of the generators does not necessarily equal 50%, and is basically determined by the network nodal equations. Therefore, according to Table IV, under the proposed method, 4015 \$/h (50%) of 8030 \$/h overall network cost is allocated to the generators irrespective of the calculation reference side; whereas under the Z-bus method, these values becomes 2568.4 \$/h (32%) and 2344.1\$/h (29%) of 8030 \$/h, for sending and receiving end selection of the calculation reference side, respectively. However, this is not a drawback of the proposed method, because unlike Z-bus method, the proposed method can lessen or increase the cost responsibility ratio of the generators, according to the policies and preferences of the power market.

C. Polish 2383-bus Test System

The system data is given in [41]. The polish 2383-bus system is considered to emphasize the applicability of the proposed method to very large practical systems. The network is part of the 7500 + bus European Union for the Coordination of the Transmission of Electricity (UCTE) system. The proposed method does not encounter the singularity problems of the Z-bus and PS methods to invert large scale sparse matrices. Intel Core i5-2410M 2.3 GHz with 6 GB RAM 64 bit win. 10 is used to run the program. MATLAB R2016a reported the elapsed time 21.47 s.

VIII. CONCLUSION

This paper proposes a new circuit theory based method for transmission cost allocation. Two important issues of the published circuit theory based TCA methods are addressed. First, how to mathematically treat the non-linear behavior of the power systems which is uncovered by the principle applied to divide the interaction terms. Second, how to make a comparison between the users' contributions to the network complex quantities. Accordingly, the proposed methodology utilized the equal sharing principle to divide the interaction terms, and provided a rationale for the principle using co-operative game theory. The method was also extended to develop the concept of real shares of the users on the network quantities by utilizing the orthogonal projection concept. This, in turn, made it possible to compare the contributions.

Based on the results of the sample 2-bus test system, the proposed method is applicable for all network structures, as it resolves the singularity problems of the Z-bus based methods to build the impedance matrix. As one of the most distinguish properties of the proposed method, numerical case studies

on IEEE 30-bus indicates that the counter flows are less intense by the proposed technique, compared with those of the Z-bus method. This feature is truly valuable, as intense counter-flows lead to cross subsidies, and in turn, bring about unfair competition in power markets. In addition, the proposed method is consistent as it provides tariffs with highest stability against temporal system variations, compared with Z-bus and PS methods. Unlike Z-bus method, the proposed method is less sensitive to the calculation side of the lines. The proposed method also reflects the size and the location of the users in the grid. The computational performance on Polish 2383-bus test system indicates that the proposed method is not complicated and computationally demanding even on realistic sized problems as it is based on the circuit theory. In addition, the proposed technique can be well applied in TCA of large practical power systems, as it overcomes the limitations of the Z-bus as well as the PS methods to invert the large-scale sparse matrices, as well. In summary, the proposed TCA method is deemed fair, efficient and satisfactory because it recovers the total transmission costs, reflects the actual "extent of use" of the transmission system by its users, provides locational signals, smoothes the counter flows, thus reduces the cross subsidies, and is also consistent, applicable to all network configurations and easy to implement to very large practical systems.

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